1. (a) $P \vee \text{TRUE} \Leftrightarrow \underline{\underline{\text{TRUE}}}$

Q \ P	T	F
T	T	T
F	T	F

$P \vee \cancel{Q}$ TRUE

(b) $P \vee \text{FALSE} \Leftrightarrow \underline{\underline{P}}$

Q \ P	T	F
T	T	T
F	T	F

$P \vee \cancel{Q}$ FALSE

(c) $P \wedge \text{TRUE} \Leftrightarrow \underline{\underline{P}}$

Q \ P	T	F
T	T	F
F	F	F

$P \wedge \cancel{Q}$ TRUE

(d) $P \wedge \text{FALSE} \Leftrightarrow \underline{\underline{\text{FALSE}}}$

Q \ P	T	F
T	T	F
F	F	F

$P \wedge \cancel{Q}$ FALSE

(e) $P \vee \neg P \Leftrightarrow \underline{\underline{\text{TRUE}}}$

Q \ P	T	F
T	T	T
F	T	F

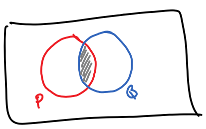
$P \vee \cancel{Q}$ $\neg P$

(f) $P \wedge \neg P \Leftrightarrow \underline{\underline{\text{FALSE}}}$

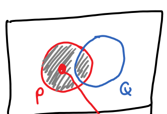
Q \ P	T	F
T	T	F
F	F	F

$P \wedge \cancel{Q}$ $\neg P$

2.



$P \wedge Q$



P

$\underline{P \wedge Q \Rightarrow P}$ BECAUSE IN EVERY CASE THAT $P \wedge Q$ IS TRUE, SO IS P (THE SHADED AREA FOR $P \wedge Q \subset$ THAT OF P)

$\underline{P \not\Rightarrow P \wedge Q}$ BECAUSE IT IS NOT TRUE THAT IN EVERY CASE THAT P IS TRUE, $P \wedge Q$ IS ALSO TRUE... SPECIFICALLY, IN THE $P \wedge \neg Q$ CASE. (THE SHADED AREA FOR $P \not\subset$ THAT OF $P \wedge Q$)

IN WORDS: IF $P \wedge Q$ IS TRUE, WE CAN CONCLUDE THAT P IS TRUE.

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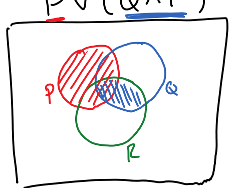
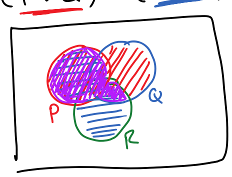
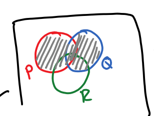
3. (a) $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$



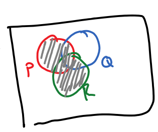
✓



$\wedge$

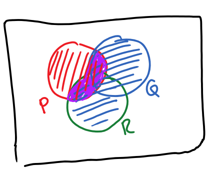


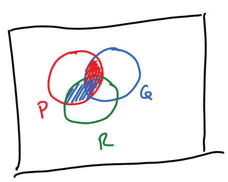
$P \vee Q$



$P \vee R$

(b) $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$





4. (a) $(P \Rightarrow Q) \wedge P \overset{\text{DEF}}{\Longleftrightarrow} (Q \vee \neg P) \wedge P \overset{\text{DISTRIBUTE}}{\Longleftrightarrow} (Q \wedge P) \vee \underbrace{(\neg P \wedge P)}_{\text{FALSE}}$

$\Longleftrightarrow (Q \wedge P) \vee \text{FALSE} \Longleftrightarrow \underline{Q \wedge P}$ → NO NEWS HERE; WE STARTED OUT KNOWING THIS!

SO IF WE KNOW ① $P \Rightarrow Q$ IS TRUE & ② P IS TRUE, WE CAN CONCLUDE THAT Q IS TRUE, TOO.

(b) $(P \Rightarrow Q) \wedge \neg Q \overset{\text{DEF}}{\Longleftrightarrow} (Q \vee \neg P) \wedge \neg Q \overset{\text{DISTRIBUTE}}{\Longleftrightarrow} \underbrace{(Q \wedge \neg Q)}_{\text{FALSE}} \vee (\neg P \wedge \neg Q)$

$\Longleftrightarrow \text{FALSE} \vee (\neg P \wedge \neg Q) \Longleftrightarrow \underline{\neg P \wedge \neg Q}$

SO IF WE KNOW ① $P \Rightarrow Q$ IS TRUE & ② Q IS FALSE, WE CAN CONCLUDE THAT P IS ALSO FALSE.

* NOTE WELL THAT $(P \Rightarrow Q) \wedge \neg P$ TELLS US NOTHING, NOR DOES $(P \Rightarrow Q) \wedge Q$.

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5. THESE ARE ALL $\vee$'S, SO WE CAN ORDER & GROUP THEM HOWEVER WE LIKE, BY COMMUTATIVITY & ASSOCIATIVITY; THERE ARE SIX ORDERINGS AND TWO WAYS TO GROUP EACH ORDERING:

$P \vee (Q \vee R)$	$P \vee (R \vee Q)$	$R \vee (P \vee Q)$	$R \vee (Q \vee P)$	$Q \vee (P \vee R)$	$Q \vee (R \vee P)$
$(P \vee Q) \vee R$	$(P \vee R) \vee Q$	$(R \vee P) \vee Q$	$(R \vee Q) \vee P$	$(Q \vee P) \vee R$	$(Q \vee R) \vee P$

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*6. $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$ → NOTE THAT WE COULD TECHNICALLY KEEP DISTRIBUTING — BUT WE'LL GET DUPLICATES ($P \wedge P$, ETC.), SO THAT IS SORT OF CHEATING!

COULD SWAP P WITH $Q \vee R$ AND/OR Q WITH R ④

COULD SWAP; $P \wedge Q$ WITH $P \wedge R$; AND/OR P WITH Q; AND/OR P WITH R. ⑧ → TOTAL: 12

7. (a) "EVERY $\underbrace{\text{REAL \#}}_{y}$ HAS A $\underbrace{\text{REAL CUBE-ROOT}}_{x}$." TRUE

(b) "EVERY $\underbrace{\text{INTEGER}}_{y}$ HAS AN $\underbrace{\text{INTEGRAL CUBE ROOT}}_{x}$" FALSE (E.G., $y=2$)

(c) $\exists\, b \in \mathbb{R}$ SUCH THAT $\forall a \in \mathbb{R}$, $b > a$ FALSE (TAKE $a = b+1$)

"SOME $\underbrace{\text{REAL \#}}_{b}$ IS LARGER THAN $\underbrace{\text{EVERY REAL \#}}_{a}$."

(d) FOR EACH $\underbrace{\text{INTEGER}}_{a}$, THERE IS A $\underbrace{\text{LARGER INTEGER}}_{b}$." TRUE (TAKE $b = a+1$)

$\forall a \in \mathbb{Z}$ $\exists\, b \in \mathbb{Z}$ SUCH THAT $b > a$

* IN (c) & (d), THE $\mathbb{Z}$ & $\mathbb{R}$ AREN'T AN ISSUE, BUT THE ORDER OF QUANTIFIERS IS!! NO SINGLE b IS LARGER THAN ALL REAL NUMBERS (PART c); BUT IF YOU'RE GIVEN AN INTEGER a FIRST (PART d), THERE IS CERTAINLY A LARGER ONE.

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8. (a)

$$\neg[Q \vee (P \wedge \neg R)] \iff \neg Q \wedge \neg(P \wedge \neg R)$$
$$\iff \neg Q \wedge (\neg P \vee \cancel{\neg\neg} R) \quad \text{CANCEL}$$
$$\iff \underline{\underline{\neg Q \wedge (\neg P \vee R)}}$$

$P \Rightarrow Q$ MEANS $Q \vee \neg P$

SO $\neg(P \Rightarrow Q)$ MEANS $(P \wedge \neg Q)$

(b) $\neg[\forall M\ \exists N$ SUCH THAT $\forall x,\ (x > N \Rightarrow 2x > M)]$

$\iff \exists M$ SUCH THAT $\neg[\exists N$ SUCH THAT $\forall x,\ (x > N \Rightarrow 2x > M)]$

$\iff \exists M$ SUCH THAT $\forall N,\ \neg[\forall x,\ (x > N \Rightarrow 2x > M)]$

$\iff \exists M$ SUCH THAT $\forall N,\ \exists x$ SUCH THAT $\neg(x > N \Rightarrow 2x > M)$

$\iff \exists M$ SUCH THAT $\forall N,\ \exists x$ SUCH THAT $x > N \wedge \neg(2x > M)$

$\iff \underline{\underline{\exists M \text{ SUCH THAT } \forall N,\ \exists x \text{ SUCH THAT } x > N \wedge 2x \leq M}}$

9. TRUE (IT WOULD CORRESPOND TO THE SET OF EVERYTHING, WHICH $\nexists$)

& $\neg$ (THE EMPTY SET CORRESPONDS TO "FALSE," AND NEGATING FALSE GIVES TRUE, WHICH IS A PROBLEM AS ABOVE!)

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10. (a) $a \in A \cup (B \setminus C) \iff a \in A \vee a \in B \setminus C$

$\iff a \in A \vee (a \in B \wedge \neg a \in C)$

I.E., $a \notin C$

(b) $y \in K \triangle (M \cap N) \iff y \in K \oplus y \in M \cap N$

$\iff y \in K \oplus (y \in M \wedge y \in N)$

(c) $x \in (X \setminus Y) \cup (X \cap Y) \iff x \in X \setminus Y \vee x \in X \cap Y$

$\iff (x \in X \wedge x \notin Y) \vee (x \in X \wedge x \in Y)$

$\iff x \in X \wedge (x \notin Y \vee x \in Y)$

$\iff x \in X \wedge \text{TRUE}$

$\iff x \in X$

DISTRIBUTIVE LAW THE OTHER DIRECTION!!

EVEN EASIER VIA VENN DIAGRAM:

(d) $A \cup B \subseteq C \iff x \in A \cup B \Rightarrow x \in C$

$\iff (x \in A \vee x \in B) \Rightarrow x \in C$

(e) $A \setminus C = C \cap B \iff [x \in A \setminus C \iff x \in C \cap B]$

$\iff [(x \in A \wedge x \notin C) \iff (x \in C \wedge x \in B)]$